Quasi-sphericals

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Previous works: Teerikorpi et al. 2015, Gramann et al. 2015, Einasto, M., et al. 2015
Definition of the superclusters

• Galaxies and clusters: high fractional overdensities up to $10^6$
• Galaxies and clusters are virialized, bound system
• Relatively homogeneous objects (size, richness, luminosity, morphology)
  • Well defined

• Superclusters a few times of the mean density
• Superclusters not virialized
• Heterogenous objects
  • Not well defined
Definition of the superclusters

- Number density filed
- Luminosity density field
- Voronoi tessellation and Wiener filtering
- Dynamical definition

What is the dynamical state of the superclusters defined with luminosity density method?
Dynamical definition

Supercluster is the volume within which the galaxies have an inward peculiar velocity component. Outside the outer surface of the supercluster, the matter is expanding along the Hubble flow.

Peculiar velocities of galaxies are available only for our local universe

Tully et al. 2014 Nature 513, 71
Global density (Planck) of the DE is about
$$\rho_\Lambda = 6 \times 10^{-30} \text{gcm}^{-3} \rightarrow \text{low but dominate because uniform across space}$$
(critical density $$\rho_{\text{crit}} = 8.52 \times 10^{-30} \text{gcm}^{-3}$$)

In standard $$\Lambda$$CDM, DE is repulsive force $$\rightarrow$$ accelerating expansion

In the regions where DE dominates over the gravitating matter structures do not grow
Since $$z=0.7$$ the formation of the structures are slowed down and halted to the supercluster scales $$\rightarrow$$ in the future they will not be necessary bound anymore
For spherically symmetric system:
The force affecting a test particle with mass $m$ as the sum of Newton’s gravity force produce by a mass $M$ and Einstein’s repulsive force due to DE (Chernin et al. 2009) when $r=2r_L$ the acceleration around the system is zero: Zero gravity radius $R_{ZG}$

$$F(R) = \left(-\frac{GM}{R^2} + \frac{8\pi G}{3}\rho_\Lambda R\right) m = \frac{4\pi}{3} GR \left(-\rho + 2\rho_\Lambda\right) m.$$ (Chernin et al. 2009)

when $\rho=2\rho_\Lambda$ the acceleration around the system is zero: Zero gravity radius $R_{ZG}$

$R_{ZG}$ is the gravitationally bound region at the present epoch. This criterion can be used define the minimum mass density for a gravitationally bound system at the present epoch.
Schematic graph of the interplay between the dark energy and the gravitational force for the spherical system with fixed mass. Different radii bound the regions according to the dominant component. Physical coordinates.

$\rho/\rho_\Lambda$ in different regions

$u=HR-v_{pec}$

$R_{ZG}$ = gravitationally bound at the present epoch

Dynamically different regions:

$R_{TR}$ = turn around radius

$R_{FC}$ = future collapse radius

$R_{ZG}$ = zero gravity radius

$R_{ES}$ = Einstein-Straus radius
\( \rho/\rho_\Lambda \) in different regions

\[ u = HR - v_{pec} \]

\( R_{\text{ZG}} \) = gravitationally bound at the present

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<td>( \rho/\rho_\Lambda )</td>
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<td>-</td>
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<td>-</td>
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<td>7.86</td>
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<td>2.36</td>
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</tr>
<tr>
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<td>2.0</td>
<td>1.40</td>
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<td></td>
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<td>2.0</td>
<td>1.46</td>
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<tr>
<td>Linear</td>
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<td>1.0</td>
<td>0.43</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>0.27</td>
<td>1.0</td>
<td>0.37</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Gramann et al. 2015
In the dynamical picture of the structures in one end we have large gravitationally unbound superclusters defined as a system with inward peculiar velocity components relative to the Hubble flow and in the other end there are clusters of galaxies in an equilibrium configuration. It is expected that between these extremes there is a continuum of structures with different dynamical states.

What is the dynamical state of superclusters?

If we know the mass and the radius of the supercluster we can apply previous approach to calculate the matter density in the volume around the system and define characteristic regions → define dynamical state of the supercluster.
Central region of the Superclusters A2142

**Fig. 1.** Distribution of galaxies in the A2142 supercluster in the sky plane in global density regions as described in the text. Red filled circles denote galaxies in the region of global density $D8 \geq 17$; yellow empty circles correspond to galaxies with global density $13 \leq D8 < 17$; blue crosses correspond to galaxies with global density $8 \leq D8 < 13$; and grey Xs galaxies with $5 \leq D8 < 8$. The size of the highest density region is approximately 1.8 degrees, and the size of the region with $D8 \geq 13$ is approximately 3 degrees; sizes in megaparsecs are given in Table 2. Number 1 marks the Abell cluster A2142, and numbers 2 and 3 indicate two regions of galaxy groups in the tail of the supercluster, as explained in the text.

**Fig. 2.** Mass corresponding to the turnaround mass $M_T(R)$ (red line), future collapse mass $M_{FC}(R)$ (violet line), zero-gravity mass $M_{ZG}(R)$ (blue line), and linear mass $M_L(R)$ (grey line; in units of $10^{15} \, h^{-1} \, M_\odot$) versus radius of a sphere $R$ in different dynamical evolution models for $\Omega_m = 0.27$. Filled circles show the total masses of galaxy groups in regions of different global density in the A2142 supercluster (Table 2). Stars denote estimated masses as explained in the text. Numbers show global density lower limit for a region (env marks Main+env region, 2 and 3 denote regions of galaxy groups in the tail of the supercluster).

Gramann et al. 2015,
Einasto et al. 2015
The useful parameter that characterizes the influence of the DE energy density ratio $<\rho_M>/\rho_\Lambda$ as calculated for the system under inspection (Teerikorpi et al. 2015).

$$\log(<\rho_M>/\rho_\Lambda) = 0.43 + \log M/10^{12} M_\odot - 3 \times \log R/{\text{Mpc}}.$$
Graph of the dark energy influence (Teerikorpi et al. 2015)

(Teerikorpi et al. 2015): the influence of DE for the system under inspection

Different regions in such graph corresponding to the mass and size of a system and its dynamical state

Location in the diagram indicates whether is overall dynamics is dominated by gravity or DE antigravity

**Fig. 1.** Log\(\langle \rho_M \rangle / \rho_\Lambda\) vs. log\(R\) for spherical systems. The inclined lines correspond to different mass values. Above the “gravity = antigravity” line, the region is dynamically dominated by gravitation, and below this line by DE. Intersections give the radii \(R_{ZV}\), \(R_{ZG}\), and \(R_{ES}\). Dotted inclined lines illustrate the case where the mass increases with the radius (see the text).

\[
\log\langle \rho_M \rangle / \rho_\Lambda = 0.43 + \log M/10^{12} M_\odot - 3 \times \log R/\text{Mpc}.
\]
Example for galaxy systems:

Different mass estimations

BUT method works only for spherical systems
Supercluster

- Liivamägi et al. 2012, luminosity density field method

- Adaptively selected catalogue.
- SDSS DR7 (Liivamägi et al. 2012)

- Total 1206 superclusters

- The volume of a supercluster is calculated from the luminosity density field as the summed number of connected grids within this supercluster. This method does not assume any specific shape for the structure, making possible a morphological analysis
Masses of the supercluster

• We approximate the supercluster masses summing together the dynamical masses of all the galaxy groups belonging to each supercluster.

• For that purpose, we first matched the group catalogue by Tago et al. (2010) with our supercluster catalogue.

\[ M_{200}^D = 7.0 \times 10^{12} \frac{R_g}{\text{Mpc}} \left( \frac{\sigma_v}{100 \text{km/s}} \right)^2 M_\odot \]

where \( \sigma_v \) is the 1D velocity dispersion and \( R_g = 4.582 \sigma_{\text{sky}} \). \( \sigma_{\text{sky}} \) is the rms deviation of the projected distance in the sky from the group center.

Tempel et al. 2014
• Following Gramann et al. (2015), (supercluster A2142), we assigned to each group having less than 5 members the median mass of such groups.

• Each supercluster contains also single galaxies. We assume that due to the survey magnitude limit every single galaxy is a member of an unresolved small group and therefore we also assigned to each galaxy the median mass of the groups having less than five members.

• In addition, as did Gramann et al. (2015), we assume that intracluster gas increases the total mass of the supercluster by 10%. 
The selection bias

- Although the total luminosities of the groups are corrected via a weighing procedure (the ratio of the expected total luminosity to the expected luminosity in the visibility window) due to the apparent magnitude limit of the sample, the group richness decreases inevitably as a function of the distance.

- Tago et al. 2010 analysis showed that the estimated group richness remains reliable up to a distance of about 300 Mpc/h. After applying this limit the number of superclusters in the original sample was reduced to 132 superclusters.

The final sample used in our analysis contains 65 superclusters.
Morphology

• While clusters and groups are, in general, relatively spherical systems, the spherical shape obviously is not the usual case for superclusters.

**Instead** the most typical shape for the supercluster is filamentary (or elongated prolate) morphology (Jaaniste et al. 1998, Basilakos 2003, Einasto et al. 2011). Also almost all adaptively selected superclusters used in our analysis are elongated at some level.
• A useful tool to study supercluster morphologies is provided shapefinder vector $K=(K_1,K_2)$ whose components $K_1$ and $K_2$ are the dimensionless products of the Minkowski geometrical quantities ($V=$volume, $S=$surface area, $C=$integrated mean curvature).

• Applied for the adaptively defined superclusters
• The ratio of the vector components $K_1/K_2 > 1$ corresponds to a pancake type structure (for an ideal pancake $K = (1, 0)$).

• The ratio $0 < K_1/K_2 < 1$ corresponds to a filament type structure (for an ideal filament $K = (0, 1)$).

• When both $K_1$ and $K_2$ are close to zero the shape is close spherical. A perfect sphere has the parameter combination $K = (0, 0)$.

• If we consider a triaxial ellipsoid with axes $a,b,c$ sphere $K = (0, 0)$ it corresponds situation $(a,b=a, c=a)$ (Sahni et al. 1998, Shandarin et al. 2004).
### TABLE 2
Deformations of a Triaxial Ellipsoid with Axes $a$, $b$, $c$

<table>
<thead>
<tr>
<th>$a$, $b$, $c$</th>
<th>$(K_1$, $K_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere $\rightarrow$ Filament</td>
<td>($0.000$, $0.000$)</td>
</tr>
<tr>
<td>(100, 100, 100)</td>
<td>($0.000$, $0.000$)</td>
</tr>
<tr>
<td>(100, 80, 80)</td>
<td>($0.004$, $0.005$)</td>
</tr>
<tr>
<td>(100, 50, 50)</td>
<td>($0.028$, $0.054$)</td>
</tr>
<tr>
<td>(100, 20, 20)</td>
<td>($0.077$, $0.300$)</td>
</tr>
<tr>
<td>(100, 10, 10)</td>
<td>($0.095$, $0.540$)</td>
</tr>
<tr>
<td>(100, 3, 3)</td>
<td>($0.100$, $0.830$)</td>
</tr>
<tr>
<td>Sphere $\rightarrow$ Pancake</td>
<td>($0.000$, $0.000$)</td>
</tr>
<tr>
<td>(100, 100, 100)</td>
<td>($0.000$, $0.000$)</td>
</tr>
<tr>
<td>(100, 100, 80)</td>
<td>($0.005$, $0.004$)</td>
</tr>
<tr>
<td>(100, 100, 50)</td>
<td>($0.054$, $0.028$)</td>
</tr>
<tr>
<td>(100, 100, 20)</td>
<td>($0.300$, $0.077$)</td>
</tr>
<tr>
<td>(100, 100, 10)</td>
<td>($0.540$, $0.095$)</td>
</tr>
<tr>
<td>(100, 100, 3)</td>
<td>($0.830$, $0.100$)</td>
</tr>
<tr>
<td>Pancake $\rightarrow$ Filament</td>
<td>($0.830$, $0.100$)</td>
</tr>
<tr>
<td>(100, 100, 3)</td>
<td>($0.830$, $0.100$)</td>
</tr>
<tr>
<td>(100, 70, 3)</td>
<td>($0.800$, $0.130$)</td>
</tr>
<tr>
<td>(100, 30, 3)</td>
<td>($0.650$, $0.330$)</td>
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<tr>
<td>(100, 10, 3)</td>
<td>($0.330$, $0.650$)</td>
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<tr>
<td>(100, 3, 3)</td>
<td>($0.100$, $0.830$)</td>
</tr>
</tbody>
</table>

K1/K2 > 1  Pancakes
0 < K1/K2 < 1  Filaments

Ideal Pancake  K1=1, K2=0
Ideal Filaments  K1=0, K2=1
Ideal Sphere  K1=0, K2=0
There is no unambiguous correspondence between the values of the shapender vectors and certain shapes but the vectors constitute a continuum of values which indicate different shapes.
Selection of the sample

• Selection bias: 1206→132 -> 65 superclusters

• The fourth Minkowski function $V_3$ describes the sum of the clumps, voids and tunnels in the system (e.g., Saar 2007). Thus a high value of the $V_3$ parameter indicates a complicated clumpy morphology, while $V_3=1$ means a structure with a single center and smooth structure.

• we require $K_1$ and $K_2$ to have values less than 0.05

• The choice of the limit 0.05 is based on our finding that the superclusters less than this value are all systems which apparently have a single dominant center and which is natural for a spherical or even pancake system.
Fig. 2. Properties of the superclusters as a function of $K1$ and $K2$. Quasi-spherical (QS1) superclusters $K1,K2 < (0.05, 0.05)$ are shown with red and green points and QS2 superclusters $K1,K2 < (0.03, 0.03)$ are shown with green symbols.
Fig. 3. Ratio $\langle \rho_M \rangle / \rho_\Lambda$ vs. log $R$/Mpc for clusters (red), quasi-spherical superclusters (green and blue), Laniakea supercluster (violet) and the central regions of the supercluster A2142

The SDSS groups as well as prominent clusters in general are usually located deep within the gravity dominated radius ($R$ much less than $R_{ZG}$), which is also a requirement for them to be virialized.

Systems above the zero-velocity radius line are totally within the collapsing region. The outer parts of systems below the line have not been retarded down to zero velocity.
Conclution

• We use topological arguments to restrict as spherical sample of superclusters as possible. We analyzed their properties and dynamical state.

• Our analysis of the quasi-spherical superclusters shows that they are relatively smaller, less massive, less luminous and they contain less galaxies and groups than other superclusters.

• Quasi-spherical objects found may be considered as the largest gravitationally dominated systems found so far.

• From the dynamical point of view these objects represent intermediate systems between clusters and the largest superclusters.

• It will be interesting to compare these results with simulations where non-virialized systems are extracted

• Due to large mass concentration in relatively small and spherical region they might be visible via SZ effect
The Laniakea supercluster (Tully et al. 2014) is not far from the E-S distance as calculated from its mass $10^{17} \text{ M}_{\text{sun}}$. 
How is the mass distributed in the universe? Does it follow, on the average, the light distribution? To address this important question, peculiar motions on large scales are studied in order to directly trace the mass distribution. It is believed that the peculiar motions (motions relative to a pure Hubble expansion) are caused by the growth of cosmic structures due to gravity. A comparison of the mass-density distribution, as reconstructed from peculiar velocity data, with the light distribution (i.e., galaxies) provides information on how well the mass traces light (see chapter by Dekel, 1994). The basic underlying relation between peculiar velocity and density is given by

\[ \nabla \cdot \vec{v} = -\Omega_m^{0.6} \delta_m = -\Omega_m^{0.6} \delta_g / b \]

https://ned.ipac.caltech.edu/level5/Sept01/Bahcall2/Bahcall11.html
• Observations:
  • Superclusters of galaxies: SDSS DR7 (Liivamägi et al. 2012), 1313 superclusters, adaptive

• Simulations:
  • Millenium simulations 1214 superclusters (Liivamägi et al. 2012)
Concluding remarks:

• Graph of dark energy significance can be used to characterize systems of galaxies.

• Different regions in the diagram correspond to these systems' dynamical state within the \( \Lambda \) dominated expanding universe.

• The study of the galaxy properties in dynamically different regions may provide interesting insight for the environmental studies of the galaxies in superclusters.

• Definition of the superclusters.

• Mass of the supercluster: \( M/L \), lensing, dynamical mass.

• Theory is for spherical systems \( \rightarrow \) the spherical superclusters \( \rightarrow \) not typical.

• Redshift/real space – Kaiser effect.

• Simulations.
Dynamical definition:

The Laniakea supercluster (Tully et al. 2014) is not far from the E-S distance as calculated from its mass $10^{17} M_{\text{sun}}$. 

Tully et al. 2014 Nature 513, 71