Quantum Gravity Phenomenology

- Lateshift and Photon orbits, observable traces of quantum gravity?

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Tartu Observatory Seminar
Tartu October 2017
1. Quantum Gravity Phenomenology

2. Dispersion Relations as Hamilton Functions

3. Symmetric Dispersion Relations

4. Observables: Cosmology

5. Observables: Spherical Symmetry

6. Conclusion and Outlook
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6. Conclusion and Outlook
Assumptions:
• There exists a consistent, yet unknown, theory of quantum gravity which describes the quantum field theoretical nature of the gravitational interaction.
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- The scale at which this quantum field theoretical of gravity becomes relevant, i.e., the non-classical nature of gravity leads to measurable effects, is the Planck Scale.

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E_{Pl} = \sqrt{\frac{\hbar c^5}{G}} \approx 1.956 \times 10^9 \text{J} \approx 1.2209 \times 10^{19} \text{GeV}
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\ell_{Pl} = \sqrt{\frac{\hbar G}{c^3}} \approx 1.616 \times 10^{-35} \text{m}
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Candidates for quantum gravity:

• String theory (fundamental particles are modes of one-dimensional strings)
• Loop quantum gravity (canonical quantisation of general relativity)
• Causal dynamical triangulation (quantise general relativity on a lattice)
• Non-commutative geometry (use non-commuting coordinates on spacetime)
• …
Aim:

- Without knowing the fundamental theory of quantum gravity one seeks to describe possible observable effects which emerge from the fundamental theory as perturbations of the classical theory
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Possible effects:
• Modifications of the dynamics of general relativity (f(R,\Phi) theories, …)
• Variations of the constants of nature (c, G, \alpha, …)
• Violations or modifications of Lorentz invariance
• Modified dispersion relations for particle motion
• Energy dependent spacetime geometries
• …
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We probe spacetime with elementary particles, photons, neutrinos, …
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- Low energetic photons probe spacetime at larger scales
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- The larger the interaction between probe and quantum nature of gravity
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The higher the probe particle energy
the higher the influence of the quantum nature of gravity
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Effective model: Energy dependent spacetime geometry

$g_{ab}(x, \frac{E}{E_{Pl}})$
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Effective model: Energy dependent spacetime geometry

\[ g_{ab}(x, \frac{E}{E_{Pl}}) \]

Difficulties:

- Particle energy E is an observer dependent quantity.
  -> observer dependent visibility of quantum gravity?
- How to describe “energy” dependent spacetime geometry covariant?
Deformed special relativity: [Magueijo, Smolin 2004]

\[ D_{SR} (p) = \eta^{ab} p_a p_b = -m^2 \iff E^2 = \tilde{p}^2 + m^2 \]
Deformed special relativity: [Magueijo, Smolin 2004]

\[ D_{SRT}(p) = \eta^{ab} p_a p_b = -m^2 \iff E^2 = \bar{p}^2 + m^2 \]

Deformation Map \( U \)

\[ U(p) = \left(f\left(\frac{E}{E_{Pl}}\right)E, g\left(\frac{E}{E_{Pl}}\right)\bar{p}\right) \]
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Absorb deformation into metric

\[ D_{DSRT} = \tilde{\eta}^{ab}(\frac{E}{E_{Pl}}) p_a p_b \]

Invariant under deformed Lorentz transformations \( \Lambda(\frac{E}{E_{Pl}}) \)
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Christian Pfeifer, Quantum Gravity Phenomenology - Observables, Tartu 2017
Curved Momentum Space: Doubly Special Relativity [Amelino-Camelia 2008],
Relative locality [Amelino-Camelia, Freidel, Kowalski Gilkman, Smolin 2011]

\[ g = g^{ab}(p) dp_a \otimes dp_b, \quad D(p) = g^{ab}(p)p_a p_b = -m^2 \]
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- modified addition of moment

\[ (p \oplus q)_a = p_a + q_a + \ell_P \Gamma^{cd}_{\ a} p_c q_d + \ldots \]

- spacetime emerges from Legendre transformation

\[ L(x, p, \lambda) = \dot{p}_a x^a + \lambda(D(p) - m^2) \]

- the fundamental space where physics happen is momentum space
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Difficulties:
- How to determine $g(p)$?
- How are observers modelled?
- How to decompose $p$ into energy $E$ and three momentum $\vec{p}$?

\[ S[x] = \int d\tau L(x, \dot{x}) \Rightarrow \frac{d}{d\tau} \frac{\partial}{\partial \dot{x}^a} L - \frac{\partial}{\partial x^a} L = 0 \]

• a homogeneous Lagrangian determines particle motion

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- L often not smooth on the light-cone
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Solved: [CP 2013]

The Finsler spacetime framework
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Dispersion Relations as Hamilton Functions

The local Lorentz invariant dispersion relation of a free point particle is

\[-E^2 + p_\alpha p_\beta \delta^{\alpha\beta} = -E^2 + \bar{p}^2 = -m^2\]

- $m$ is the invariant mass parameter
- $E = g(\gamma,p)$ is the energy
- $p_\alpha = g(e_\alpha,p)$ is the spatial momentum

an observer on worldline $\gamma$ associates to the particle with 4-momentum $p$
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Covariant: From observer frames to a frame independent expression

\[e_\mu = A^a_\mu \partial_a \Rightarrow p_\mu = A^a_\mu p_a\]

The frame transformations transforms the dispersion relation:

\[-m^2 = g^{ab}(x) p_a p_b = H(x, p)\]
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The covariant dispersion on a curved spacetime:

- Level set of a Hamilton function on phase space
- determines particle motion via Hamilton equations of motion
- determines the curved geometry of phase space, momentum space and spacetime (Hamilton geometry) [Barcaroli 2015]
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**General Principle:**

Four momentum dependent spacetime and momentum space geometry modelled by Hamilton function \(H(x, p)\)

\[m^2 = g_{ab}(x)p^a p^b = H(x, p)\]

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Symmetries of Dispersion Relations

Symmetries: Diffeomorphisms on phase space which leave $H$ invariant

\[ H(\Phi(x, p)) = H(x + \xi, p + \bar{\xi}) = H(x, p) \]
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$$H(\Phi(x, p)) = H(x + \xi, p + \bar{\xi}) = H(x, p)$$

Infinitesimal this corresponds to: $H$ is constant along a vector field $X$

$$X(H) = \xi^a(x, p) \partial_a H + \bar{\xi}_a(x, p) \bar{\partial}^a H = 0$$

[Barcaroli, Brunkhorst, Gubitosi, Loret, CP, 2016]
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Manifold induced symmetries are defined by special vector fields $X^C$

$$X^C(H) = \xi^a(x)\partial_a H - p_q\partial_a \xi^q(x)\bar{\partial}^a H = 0$$
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These are complete lifts of vector fields $X$ defining infinitesimal
diffeomorphisms on spacetime

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Each manifold induced symmetry gives rise to a constant of motion

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Symmetries of Dispersion Relations

Symmetries: Diffeomorphisms on phase space which leave $H$ invariant

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Manifolds induced symmetries are defined by special vector fields $X$ defining infinitesimal diffeomorphisms on spacetime

The generalisation of the Killing equation determines spacetime symmetric Hamiltonians

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Homogeneous and isotropic dispersion relations

Homogeneity and isotropy is induced by the vector fields

\[ X_1 = \sin \phi \partial_\theta + \cot \theta \cos \phi \partial_\phi \]
\[ X_2 = -\sin \phi \partial_\theta + \cot \theta \sin \phi \partial_\phi \]
\[ X_3 = \partial_\phi \]
\[ X_4 = \chi \sin \theta \cos \phi \partial_r + \frac{\chi}{r} \cos \theta \cos \phi \partial_\theta - \frac{\chi}{r \sin \theta} \sin \phi \partial_\phi \]
\[ X_5 = \chi \sin \theta \sin \phi \partial_r + \frac{\chi}{r} \cos \theta \sin \phi \partial_\theta + \frac{\chi}{r \sin \theta} \cos \phi \partial_\phi \]
\[ X_6 = \chi \cos \theta \partial_r - \frac{\chi}{r} \sin \theta \partial_\theta \]
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The symmetry conditions \( X_I^C(H) = 0 \) yields

\[ H(x, p) = H(t, p_t, w), \quad w^2 = (1 - kr^2)p_r^2 + \frac{1}{r^2} p_{\theta}^2 + \frac{1}{r^2 \sin \theta^2} p_{\phi}^2 \]
Spherically Symmetric Dispersion Relations

Spherical symmetry is induced by the vector fields

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The symmetry conditions \( X^c_1(H) = 0 \) yields

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The most general homogeneous and isotropic Hamiltonian

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Massless Particle Dynamics

The most general homogeneous and isotropic Hamiltonian

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Hamilton equations of motion for curves: \((t(\tau), r(\tau), \Theta = \frac{\pi}{2}, \phi = 0)\)

\[ \dot{p}_t = -\partial_t H \]
\[ \dot{p}_r = \partial_w H \frac{1}{w} kr p_r^2 \]
\[ \dot{p}_\theta = 0 \]
\[ \dot{p}_\phi = 0 \]
\[ \dot{t} = \bar{\partial}_t H \]
\[ \dot{r} = \partial_w H \frac{1}{w} \chi^2 p_r \]
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Constant of motion

\[ w^2 = \text{const} = \chi p_r^2 \]

Two remaining equations of interest

\[ H(t, p_t, w) = 0 \iff p_t(t, w) \]
\[ r'(t) = \frac{dr}{dt} = \frac{\dot{r}}{t} = \frac{1}{w} \chi^2 p_r \frac{\partial_w H}{\partial_t} \]
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Redshift:

\[ z(t_i, t_f) \equiv \frac{p_t(t_i) - p_t(t_f)}{p_t(t_f)} \]
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Lateshift:

\[ r^{\text{hard}}(0) = r^{\text{soft}}(0) = 0 \]
Observables: Redshift and Lateshift

Two remaining equations of interest

\[ H(t, p_t, w) = 0 \iff p_t(t, w) \]

\[ r'(t) = \frac{dr}{dt} = \frac{\dot{r}}{t} = \frac{1}{w} \chi^2 p_r \frac{\partial w H}{\partial t} \]

Redshift:

\[ z(t_i, t_f) = \frac{p_t(t_i) - p_t(t_f)}{p_t(t_f)} \]

Lateshift:

\[ r^{\text{hard}}(0) = r^{\text{soft}}(0) = 0 \]

\[ R = r^{\text{hard}}(t^{\text{hard}}) = r^{\text{soft}}(t^{\text{soft}}) \]

\[ \Delta t = t^{\text{hard}} - t^{\text{soft}} \]
Observables: Redshift and Lateshift

Two remaining equations of interest

\[ H(t, p_t, w) = 0 \iff p_t(t, w) \]

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\[ H_{qFLRW} = -\frac{4}{\ell^2} \sinh \left( \frac{\ell}{2} p_t \right)^2 + a(t)^{-2} e^{\ell p_t w^2} \]

Redshift:

\[ z(t_i, t_f) = - \frac{\ell p_t(t_i)}{\ln \left( 1 - \frac{a(t_i)}{a(t_f)} (1 - e^{-\ell p_t(t_i)}) \right)} - 1 \]

\[ = \left( \frac{a(t_f)}{a(t_i)} - 1 \right) \left( 1 + \frac{\ell}{2} p_t(t_i) \right) + O(\ell^2) \]

Lateshift:

\[ \Delta t \big|_{a(t) = e^{ht}} = t^{\text{hard}} - t^{\text{soft}} \]

\[ = -\frac{\ell}{\hbar} p_t(t^{\text{hard}}) \left( z + \frac{z^2}{2} \right) \]
Observables: Redshift and Lateshift

Two remaining equations of interest:

\[
H(t, p_t, w) = 0 \iff p_t(t, w) \quad r'(t) = \frac{dr}{dt} = \frac{\dot{r}}{t} = \frac{1}{w} \chi^2 p_r \frac{\partial_w H}{\partial_t} \\
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\]

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z(t_i, t_f) = -\frac{\ell p_t(t_i)}{\ln \left( \frac{1 - \frac{a(t_i)}{a(t_f)} (1 - e^{-\ell p_t(t_i)})}{1 - \frac{a(t_i)}{a(t_f)}} \right)} - 1
\]

Lateshift:

Photons of high energy are slower than low energetic photons due to their stronger interaction with the quantum nature of gravity.

\[
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Observables: Redshift and Lateshift

Two remaining equations of interest

\[
H(t, p_t, w) = 0 \Leftrightarrow p_t(t, w) \\
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4-momentum dependent spacetime geometry

\[
\Delta t|_{a(t) = e^{ht}} = t^{hard} - t^{soft}
\]

\[
= -\frac{\ell}{\hbar} p_t(t^{hard}) \left( z + \frac{z^2}{2} \right)
\]
Lateshift from Ice Cube Data

9 GRB-neutrino candidates
Blue: Late neutrinos
Black: Early Neutrinos

11 Photon candidates
for energy dependent time of arrival

Lateshift with respect to a particle not subject to modified dispersion relation

Graphics:

“In-vacuo-dispersion features for GRB neutrinos and photons”
Amelino-Camelia, D’Amico, Rosati, Loret; Nature Astronomy 1 (2017)
arXiv:1612.02765
Lateshift from Ice Cube Data

Photons and Neutrinos combined

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1. Quantum Gravity Phenomenology

2. Dispersion Relations as Hamilton Functions

3. Symmetric Dispersion Relations

4. Observables: Cosmology

5. Observables: Spherical Symmetry

6. Conclusion and Outlook
The most general static spherically symmetric Hamiltonian

\[ H(x, p) = H(x, r, p_t, p_r, w), \quad w^2 = p_{\theta}^2 + \frac{1}{\sin \theta^2} p_{\phi}^2 \]
Particle Dynamics

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Hamilton equations of motion for circular curves: \((t(\tau), r = R, \Theta = \frac{\pi}{2}, \phi(\tau))\)

\[
\begin{align*}
\dot{p}_t &= 0 \\
\dot{p}_r &= -\partial_r H \\
\dot{p}_\theta &= 0 \\
\dot{p}_\phi &= 0 \\
\dot{t} &= \bar{\partial}_t H \\
\dot{r} &= 0 \\
\dot{\theta} &= 0 \\
\dot{\phi} &= \bar{\partial}_\phi H
\end{align*}
\]

Constant of motion

\[ E = p_t, \quad \mathcal{L} = p_\phi = w \]
Particle Dynamics

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\dot{p}_\theta &= 0 & \dot{\theta} &= 0 \\
\dot{p}_\phi &= 0 & \dot{\phi} &= \bar{\partial}_\phi H
\end{align*}
\]

Constant of motion

\[ E = p_t, \quad \mathcal{L} = p_\phi = w \]

Three remaining equations of interest

\[ H(r, p_t, p_r, w) = 0 \]

\[
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\end{align*}
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Innermost Circular Photon Orbits:

Strategy:

1.) \[ 0 = \bar{\partial}_r H \Rightarrow p_r(R, E, \mathcal{L}) \]

2.) \[ 0 = -\bar{\partial}_r H \Rightarrow R(E, \mathcal{L}) \]
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\( \kappa \)-Poincaré Schwarzschild Hamiltonian:

\[
H(x, p) = -\frac{4}{\ell^2} \sinh \left( \frac{\ell}{2} \frac{p_t}{\sqrt{1 - \frac{r_s}{r}}} \right)^2 + e \sqrt{1 - \frac{r_s}{r}} \left( \left(1 - \frac{r_s}{r}\right)p_r^2 + \frac{1}{r^2} \omega^2 \right)
\]
Three remaining equations of interest

\[ H(r, p_t, p_r, w) = 0 \]

\[ \dot{p}_r = -\partial_r H \quad 0 = \dot{r} = \bar{\partial}_r H \]

**Innermost Circular Photon Orbits:**

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**κ-Poincaré Schwarzschild Hamiltonian:**

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\[ r_m=0 = \frac{3}{2} r_s + \frac{\ell w}{6} \]
Three remaining equations of interest

\[ H(r, p_t, p_r, w) = 0 \]

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**Innermost Circular Photon Orbits:**

**Strategy:**

1. \( 0 = \partial_r H \Rightarrow p_r(R, E, \mathcal{L}) \)

**Photons of larger angular momentum orbit a black hole at larger distance**

**4-momentum dependent spacetime geometry**

\[ H(x, p) = -\frac{4}{\ell^2} \sinh \left( \frac{\ell}{2} \frac{p_t}{\sqrt{1 - \frac{r_s}{r}}} \right) + e^{\sqrt{1 - \frac{r_s}{r}}} \left( (1 - \frac{r_s}{r})p_r^2 + \frac{1}{r^2}w^2 \right) \]

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The Idea: The higher the probe particle energy, the higher the influence of the quantum nature of gravity.
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The covariant dispersion on a curved spacetime:
- Level set of a Hamilton function $H(x,p)$ on phase space
- determines the 4-momentum dependent curved geometry of momentum space and spacetime (Hamilton geometry)
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### Summary

The Idea: The higher the probe particle energy the higher the influence of the quantum nature of gravity.

The covariant dispersion on a curved spacetime:
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The generalisation of the Killing equation determines spacetime symmetric Hamiltonians.

**Late-shift**: Photons of high energy are slower than low energetic photons due to their stronger interaction with the quantum nature of gravity.

**Photon Orbits**: Photons of larger angular momentum orbit a black hole at larger distance.
Modell independent first order perturbation of GR

\[ H(x, p) = g^{ab}(x)p_ap_b + \epsilon h(x, p) \]

The locally \( \kappa \)-Poincaré Hamilton

\[ H_{Zg} = -\frac{4}{\ell^2} \sinh \left( \frac{\ell}{2} Z(P) \right)^2 + e^{\ell Z(P)} \left( g^{-1}(P, P) + Z(P)^2 \right) \]
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Phenomenology

- redshift/lateshift
- lensing
- horizons/thermodynamics of black holes
- rotation curves of galaxies
- classical tests of special relativity: Michelson-Moreley, Kennedy-Thorndike, Ives-Stilwell
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- Hopf Algebra symmetries of modified dispersion relations
- singularity theorems

Curved Spacetimes with local κ-Poincaré symmetry arXiv: 1703.02058
Planck-scale-modified dispersion relations in homogeneous and isotropic spacetimes arXiv: 1612.01390
Thank you for your attention

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